Hydrodynamic calculation Butterfly valve (lenticular disc)

The butterfly valve is a relatively old and proven type of valve that is most often used to close the water dam pipeline. From the butterfly valve, initially, it was only required to withstand the static water pressure and seal perfectly in the closed position. Today it is required from the butterfly valve to be able to safely close the piping at maximum flow. When closing from an open position, the disc resists the current flowing. The flow is very complex in this case and in certain opening positions the valve is extensively stressed by hydrodynamic forces. The dynamic effects of the flow are manifested by a pulsating force acting in the front of the disc eccentrically with respect to the axis of rotation. As a result, it acts on the disc of the hydrodynamic torque in the sense of closing. The resulting hydrodynamic force in the butterfly valve may, in some positions, be greater than the force which causes a static water pressure on the valve in the closed position.

In order to reduce the water hammer in the piping, it is very important to solve the most appropriate course of closure of the butterfly valve. A very important basis for calculating the water hammer is the characteristic flow and pressure losses in the valve. The dynamic effects of the water stream are substantially dependent on the hydraulic ratios in the pipeline and the valve. The flow of water without cavitation is different from the flow.
with cavitation or aeration of the space behind the valve. In the stage of fully developed cavitation there is a strong vibration, noise, great pulsation of hydrodynamic forces and torque. Shocks and vibrations of the butterfly valve can be transferred to other constructions parts (vibrations of foundations). In adverse cases, resonance of different origins can occur, and thus a serious threat to safety.

We need to know not only the mean values of the hydrodynamic forces and torque values, but also the pulsation, the maximum amplitude, when designing the butterfly valve and the connecting pipe. We also need to fully understand the effect of cavitation, the effect of aeration and the effect of the different placement of the valve in the pipeline on its hydraulic and dynamic characteristics.
1. Calculation of pressure on the butterfly valve during its rapid closure:

To calculate the pressure on the butterfly valve, we need to know the rated net head at the zero flow (closed valve). Increase pressure on water hammer must be calculated before the hydrodynamic calculation of the butterfly valve. The maximum flow rate must be defined in the open position, which must be closed safely. A must be a defined aerated space behind the valve due to the under-pressure behind the valve.

To calculate the pressure on the butterfly valve because of the ignorance of the piping system, a calculation for a simple serial connection of the control valves (variable and constant resistance) will be used.

**Relative Flow:**

\[ Q_{p\alpha} = \frac{f_{r\alpha}}{\sqrt{p + f_{r\alpha}^2(1 - p)}} \]

- \( Q_{p\alpha} \): relative flow
- \( f_{r\alpha} \): reduced free flow area in the throttle control system
- \( p \): pressure parameter

**Reduced free flow area in the throttle control system:**

\[ f_{r\alpha} = \frac{K_{Q\alpha}}{K_{Q_{\text{max}}}} \]

- \( f_{r\alpha} \): reduced free flow area in the throttle control system
- \( K_{Q\alpha} \): flow coefficient in position \( \alpha \)
- \( K_{Q_{\text{max}}} \): max. flow coefficient

**Pressure parameter:**

\[ p = \frac{\Delta h}{h_0} \]

- \( p \): pressure parameter
- \( \Delta h \): theoretical pressure in the closure at full opening [m]
- \( h_0 \): rated net head [m]

**Theoretical pressure in the closure at full opening:**

\[ \Delta h = \frac{v_0^2}{2g} \times (\zeta + 1) \]

- \( \Delta h \): theoretical pressure in the closure at full opening [m]
- \( v_0 \): valve speed [m/s]
- \( g \): gravitational acceleration [m/s²]
- \( \zeta \): local loss factor for open valve
Valve speed:
\[ v_0 = \frac{4Q_{max}}{\pi D^2} \]

\( v_0 \) valve speed \quad \text{[m/s]}
\( Q_{max} \) flow \quad \text{[m}^3/\text{s]}
\( D \) valve diameter \quad \text{[mm]}

Flow in pipeline:
\[ Q_\alpha = Q_{p\alpha} \times Q_{max} \]

\( Q_\alpha \) flow in pipeline in position \( \alpha \) \quad \text{[m}^3/\text{s]}
\( Q_{p\alpha} \) relative flow \quad \text{[]}
\( Q_{max} \) flow \quad \text{[m}^3/\text{s]}

The water speed in the pipeline:
\[ v_\alpha = \frac{4Q_\alpha}{\pi D^2} \]

\( v_\alpha \) the water speed in the pipeline in position \( \alpha \) \quad \text{[m/s]}
\( Q_\alpha \) flow in pipeline in position \( \alpha \) \quad \text{[m}^3/\text{s]}
\( D \) valve diameter \quad \text{[mm]}

The pressure loss in the pipeline:
\[ H_{L\alpha} = \frac{v_\alpha^2}{2g} \zeta_\alpha \]

\( H_{L\alpha} \) the pressure loss in the pipeline in position \( \alpha \) \quad \text{[m]}
\( v_\alpha \) the water speed in the pipeline in position \( \alpha \) \quad \text{[m/s]}
\( g \) gravitational acceleration \quad \text{[m/s}^2]\]
\( \zeta_\alpha \) loss factor in position \( \alpha \) \quad \text{[]}

Loss factor:
\[ \zeta_\alpha = \frac{1 - K_{Q\alpha}^2}{K_{Q\alpha}^2} \]

\( \zeta_\alpha \) loss factor in position \( \alpha \) \quad \text{[]}
\( K_{Q\alpha} \) flow coefficient in position \( \alpha \) \quad \text{[]}

Pressure on the butterfly valve:
\[ H_{v\alpha} = H_{L\alpha} + \frac{v_\alpha^2}{2g} + (1 - Q_{p\alpha}) \times (\Delta P + P_{atm}) \]

\( H_{v\alpha} \) pressure on the butterfly valve in position \( \alpha \) \quad \text{[m]}
\( H_{L\alpha} \) the pressure loss in the pipeline in position \( \alpha \) \quad \text{[m]}
\( v_\alpha \) the water speed in the pipeline in position \( \alpha \) \quad \text{[m/s]}
gravitational acceleration [m/s²]
relative flow []
increasing pressure on water hammer [m]
under-pressure behind the valve [m]

Relative flow:
To calculate the increase in water hammer pressure.

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2. Accuracy of measurements:

Line pressure was measured on vertical U-tubes with fill, limit relative error in pressure measurement:
\[ \delta_p \leq \pm 1\% \]

When measuring water flow using a calibrated Venturi tube, the limit relative error was not higher as:
\[ \delta_q \leq \pm 1,5\% \]

When measuring the flow of air through a calibrated gas meter, the limit relative error was not higher as:
\[ \delta_{qa} \leq \pm 2\% \]

The surface of the pipeline was made with a limit relative error:
\[ \delta_r = \pm 1,2\% \]

Limit relative medium speed error \( v \):
\[ \delta_v = \delta_q + \delta_F = \pm (1,5 + 1,2) = \pm 2,7\% \]

Limit relative error of the flow coefficient \( \mu \):
\[ \delta_\mu = \delta_q + \delta_F + \frac{1}{2} \delta_v = \pm \left( 1,5 + 1,2 + \frac{1}{2} * 1 \right) = \pm 3,2\% \]

Limit relative error of cavitation factor \( \sigma \):
\[ \delta_\sigma = \pm 2,7\% \]

Pressure diaphragm sensors and electrical devices that were used to measure hydrodynamic forces and torques were the limit relative error:
\[ \delta_{Fx} = \delta_{Fy} = \delta_m = \pm 2\% \]

Limit relative pipe diameter error:
\[ \delta_D = \pm 0,625\% \]

Limit relative torque coefficient error \( k_m \):
\[ \delta_{k_m} = \delta_m + 3 \delta_D + \delta_p = \pm (2 + 3 * 0,625 + 1) = \pm 4,9\% \]

Limit relative error of the hydrodynamic force coefficient \( k_x, k_y \):
\[ \delta_{k_x} = \delta_{k_y} = \delta_{Fx} + \delta_F + \delta_p = \pm (2 + 1,2 + 1) = \pm 4,2\% \]

Limit relative error of maximum amplitude measured by pulses:
\[ \delta_A = \pm 8\% \]

Limit relative error of the coefficients of maximum amplitudes of pulsations of hydrodynamic forces:
\[ \delta_{ax} = \delta_{ay} = \delta_A + \delta_F + \delta_p = \pm (8 + 1,2 + 1) = \pm 10,2\% \]
Limit relative error of the max. torque amplitude factor:
\[ \delta_{\alpha_m} = \delta_A + 3\delta_D + \delta_P = \pm(8 + 3 \times 0.625 + 1) = \pm 10.9\% \]

Limit relative error of Froude number \( F_r \):
\[ \delta_{F_r} = \delta_v + 0.5\delta_D = \pm(2.7 + 0.5 \times 0.625) = \pm 3\% \]

Limit relative error of aeration factor \( \beta \):
\[ \delta_{\beta} = \delta_{Q_A} + \delta_Q = \pm(2 + 1.5) = \pm 3.5\% \]
3. Guideline for the use of the hydrodynamic characteristics of the butterfly valve:

In the annex section of Fig. 2 to 16, charts of dimensionless coefficients are constructed. These graphs are the basis for constructing the hydrodynamic characteristics of the butterfly valve for the projected water dam.

**Angle between pipe axis and hydraulic force:**

\[ \phi_\alpha = \tan^{-1} \left( \frac{K_{y\alpha}}{K_{x\alpha}} \right) \times \frac{180}{\pi} \]

- \( \phi_\alpha \) angle between pipe axis and hydraulic force in position \( \alpha \) [°]
- \( K_{y\alpha} \) coefficient of hydraulic force on a disc in the axis \( y \) in position \( \alpha \) [ ]
- \( K_{x\alpha} \) coefficient of hydraulic force on a disc in the axis \( x \) in position \( \alpha \) [ ]

**Cavitation number:**

\[ \sigma_\alpha = \frac{10 - 0,1 + h_0 - H_{L\alpha}}{H_{v\alpha}} \]

- \( \sigma_\alpha \) cavitation number [ ]
- \( h_0 \) rated net head [m]
- \( H_{L\alpha} \) the pressure loss in the pipeline in position \( \alpha \) [m]
- \( H_{v\alpha} \) pressure on the spherical valve in position \( \alpha \) [m]

**Forces on disc in axis \( x \):**

\[ F_{x\alpha} = \frac{\pi D^2}{4} \times \rho \times g \times H_{v\alpha} \left( \left( 1 \pm \delta_{k_x} \right) \times K_{x\alpha} \right) \pm \left( 1 + \delta_{a_x} \right) \times a_{x\alpha} \]

- \( F_{x\alpha} \) forces on disc in axis \( x \) in position \( \alpha \) [kN]
- \( D \) valve diameter [mm]
- \( \rho \) density of liquid [Kg/m³]
- \( g \) gravitational acceleration [m/s²]
- \( H_{v\alpha} \) pressure on the butterfly valve in position \( \alpha \) [m]
- \( \delta_{k_x} \) limit relative error of the hydrodynamic force coefficient [ ]
- \( \delta_{a_x} \) limit relative error of the coefficients of maximum amplitudes of pulsations of hydrodynamic forces [ ]
- \( K_{x\alpha} \) coefficient of hydraulic force on a disc in the axis \( x \) in position \( \alpha \) [ ]
- \( a_{x\alpha} \) the amplitude of the hydraulic force to the axis \( x \) disc in position \( \alpha \) [ ]

**Forces on disc in axis \( y \):**

\[ F_{y\alpha} = \frac{\pi D^2}{4} \times \rho \times g \times H_{v\alpha} \left( \left( 1 \pm \delta_{k_y} \right) \times K_{y\alpha} \right) \pm \left( 1 + \delta_{a_y} \right) \times a_{y\alpha} \]

- \( F_{y\alpha} \) forces on disc in axis \( y \) in position \( \alpha \) [kN]
- \( \delta_{k_y} \) limit relative error of the hydrodynamic force coefficient [ ]
- \( \delta_{a_y} \) limit relative error of the coefficients of maximum amplitudes of pulsations of hydrodynamic forces [ ]
- \( K_{y\alpha} \) coefficient of hydraulic force on a disc in the axis \( y \) in position \( \alpha \) [ ]
- \( a_{y\alpha} \) the amplitude of the hydraulic force to the axis \( y \) disc in position \( \alpha \) [ ]
Forces on disc:

\[ F_α = \sqrt{F_{xα}^2 + F_{yα}^2} \]

\( F_{xα} \) forces on disc in position \( α \) [kN]
\( F_{yα} \) forces on disc in position \( α \) [kN]

Hydraulic moment:

\[ M_α = D^3 * ρ * g * H_{va} \left( \left( 1 \pm \delta_{km} \right) * K_{ma} \right) \pm \left( 1 + \delta_{am} \right) * a_{mA} \]

\( \delta_{km} = 0,049 \)
\( \delta_{am} = 0,109 \)
4. Dimensioning aerated hole:

To reduce valve vibration, pulsation of hydrodynamic forces and erosion effects of cavitation by aerating the area behind the valve. The aerated hole should be large enough for air flow to reach  

$$Q_{air} = 0.2Q_\infty$$

according to research results. It is subject to the condition

$$\frac{L+vp}{v+ceff} < 50000$$

otherwise  

$$Q_{air} = max[Q_{max} - Q_\infty; 0.2Q_\infty]$$

The aerated hole must be placed on the lateral side behind the valve seat at a distance of 0.5D. The disc must be closed in the clockwise direction and the flow through the valve must be left to right, see fig.1.

Average water velocity in narrow cross section:

$$v_x = \frac{Q}{\pi D^2/4 (1 - \sin \alpha)}$$

$$v_x$$  average water velocity in narrow cross section  [m/s]  

$$Q$$  flow  [m³/s]  

$$D$$  valve diameter  [mm]  

$$\alpha$$  the angle of opening of the disc from the open position  [°]  

Froude number:

$$\sqrt{Fr} = \frac{v_x}{\sqrt{g \frac{D}{2} \frac{1 - \sin \alpha}{3 + \sin \alpha}}}$$

$$\sqrt{Fr}$$  Froude number  []  

$$v_x$$  average water velocity in narrow cross section  [m/s]  

$$g$$  gravitational acceleration  [m/s²]  

$$D$$  valve diameter  [mm]  

$$\alpha$$  the angle of opening of the disc from the open position  [°]  

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Coefficient of under-pressure of aerated hole:

$$f_2 = f * v_x^2/v^2$$

$$f_2$$  coefficient of under-pressure of aerated hole  []  

$$v_x$$  average water velocity in narrow cross section  [m/s]  

$$v$$  speed in pipeline  [m/s]  

Effective closing time factor:

$$c_{ef} = \min \left[ \lim_{\alpha \to 70} \frac{1}{9\left[Q_{p\alpha} - Q_{p\alpha+10}\right]} \right]$$

$$c_{ef}$$  effective closing time factor  []
$Q_{pn}$  relative flow in position n

**Under-pressure behind the valve:**

$$P_{2air} = \min \left\{ 1 \times 10^5; f_2 \frac{v^2}{2g} \rho + (1 - Q_{pa}) \min \left( \frac{L \cdot v \cdot \rho}{t \cdot c_{ef}}; 1 \times 10^5 \right) \right\}$$

- $P_{2air}$  under-pressure behind the valve  [Pa]
- $f_2$  coefficient of under-pressure of aerated hole  []
- $v$  speed in pipeline  [m/s]
- $g$  gravitational acceleration  [m/s$^2$]
- $\rho$  density of liquid  [Kg/m$^3$]
- $Q_{pa}$  relative flow  []
- $L$  the length of the pipeline behind the valve  [m]
- $t$  valve closing time  [s]
- $c_{ef}$  effective closing time factor  []

**Air velocity:**

Air velocity in the narrowest cross section

$$v_{air} = \min \left\{ 0.7 \sqrt{\frac{2 \cdot P_{2air}}{\rho_{air}}}; 250 \right\}$$

- $v_{air}$  air velocity  [m/s]
- $P_{2air}$  under-pressure behind the valve  [Pa]
- $\rho_{air}$  air density  [Kg/m$^3$]

**Air flow area of the aerated hole:**

The minimum flow area of the aerated hole is located in the shell of the butterfly valve. Air flow area of the aerated hole need not be one, but there may be several. To calculate the area of the aerated pipeline the air velocity should not exceed $v_{air} = 50$ m/s

$$f_{air} = \frac{Q_{air}}{v_{air}}$$

- $Q_{air}$  air flow via the aerated hole  [m$^3$/s]
- $f_{air}$  the flow area of the aerated hole  [m$^2$]
- $v_{air}$  air velocity  [m/s]

When calculating the aerated hole, the ability of the aerated device to assess whether it meets all the under-pressure and air flow rates. At low pressure parameters $p<0.2$ there may be a small under-pressure behind the valve that the aerated device may not be functional and therefore the hydrodynamic calculation must be calculated without aerated.
5. Conclusion:
The butterfly valve must be hydraulically positioned in a straight diameter D. The effect of cavitation on pressure losses, flow and dynamic effects of the water stream does not occur immediately in the initial stage (when the first steam bubbles are formed), but only with a fully developed cavitation.
When aerated the water stream in the pipeline beyond the valve, the pressure loss in the valve increases, but the mean values of hydrodynamic forces and torques are reduced. With aerated, the pulsation of hydrodynamic forces, torque pulsation, and pulse pressure in the piping before and after the valve will greatly reduce.

Literature:
Miroslav Žajdlík: Dynamické účinky vodného prúdu na klapkové uzávery v potrubí 1967
Miroslav Nechleba: Vodní turbíny jejich konstrukce a příslušenství 1954
V. Kolář, St. Vinopal: Hydraulika průmyslových armatur 1963
Cavitation number

Fig. 2
Coefficient of discharge $\sigma_{ini}$

$\alpha \left[ ^\circ \right]$
Coefficient of hydraulic torque $\sigma_\text{ini}$
Coefficient of force $\sigma_{ini}$

Fig. 5
Coefficient of discharge $\sigma_{dev}$

$\alpha$ [°]

Fig. 6
Coefficient of hydraulic torque $\sigma_{dev}$

Fig. 7
Coefficient of force $\sigma_{dev}$

$K_x$

$K_y$

$\alpha [\degree]$
Coefficient of discharge $\sigma_{\text{min}}$

$\alpha$ [°]

Fig.9
Coefficient of hydraulic torque $\sigma_{\text{min}}$

$\alpha \, [^\circ]$
Coefficient of force $\sigma_{\text{min}}$

$K_x$  
$K_y$

Fig. 11
Coefficient of discharge $\beta=0.2$

Fig. 12
Coefficient of hydraulic torque $\beta=0.2$
Coefficient of force $\beta=0,2$

Fig. 14
Fig. 15
Amplitude coefficient $a_x$ and $a_y$

$\alpha$ [$^\circ$]

Fig. 16